DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

M.A/M.Sc. 2nd Semester

Name of Programme : M.A/M. Sc. Mathematics
Paper Type : Theory
Paper Code : MAT-508
Paper Title : Topology-II
Full Marks : 80
Pass Marks : 32 **Duration: 3 Hours**

The figures in the margin indicate full marks for the questions

Answer all the questions:

Answer any three from the following questions: $10 \times 3 = 30$

1. a) Define a Hausdorff space and locally connected spaces.

   b) Prove that a nonempty subset X of the real line R is connected if and only if it is an interval. 2+8=10

2. State and prove Urysohn Metrisation Theorem. 10

3. Prove that the product space of two

   a) T_0 space is T_0 space.

   b) T_1 space is T_1 space.

   c) Hausdorff spaces is a Hausdorff space. 3+3+4=10

4. a) Define first axiom of countability, second axiom of countability and normal spaces.

   b) Let $(X, \mathfrak{I}_1)$ and $(X, \mathfrak{I}_2)$ be two topological spaces. Prove that the product space $(X \times Y, \mathfrak{I})$ is connected if and only if X and Y are connected. 3+7=10

Answer any three from the following questions: $10 \times 3 = 30$

5. a) Define net, subnet and filter.

   b) State and prove Tube Lemma.

   c) State and prove Tychonoff Theorem. 3+3+4=10

6. State Bolzano-Weierstrass property. Prove that a space X is Hausdorff if and only if each net in X converges to atmost one point in X. 1+9=10

7. Define compact spaces. Let X be a Hausdorff space. Prove that
   a) A compact subset of X is closed.
   b) Any two disjoint compact subsets of X have disjoint nbds. 1+4+5=10

8. Define countabily compact. Prove that a T_1 space X is countably compact if and only if it has the Bolzano-Weistrass property. 1+9=10

Answer any two from the following questions: **$10 \times 2 = 20$**

9. a) Define locally finite, homeomorphism, open function, closed function and paracompact.
   b) Prove that the product of a paracompact space and a compact space is paracompact. 5+5=10

10. Define Homotopy map. State and prove the Fundamental Theorem of Algebra. 1+9=10

11. Define locally metrisable. State and prove Ngata-Smirnov metrisation theorem. 1+9=10

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